201049

M. A./M. Sc. (First Semester) Examination, Dec. 2021

(For Regular/ATKT/Ex. Students)

MATHEMATICS

Paper : Second

(Real Analysis)

Time Allowed : Three hours

Maximum Marks : 42

Note : *Attempt all questions of all sections as directed.*

Section-'A'

(Objective Type questions) 7×1=7

Note : *Attempt all questions. Each question carries 1 mark.*

1. Choose the correct option :

(i) If $f \in R[a, b]$ then :

(a) $\left|\int_a^b f\, dx\right| = \int_a^b |f|\, dx$

(b) $\left|\int_a^b f\, dx\right| \le \int_a^b f\, dx$

(c) $\left|\int_a^b f\, dx\right| \le \int_a^b |f(x)|\, dx$

(d) None of these

(ii) Let v be a continuously differentiable curve on $[a, b]$, then v is rectifiable and

(a) $\Delta_v(a, b) = \int_a^b |v(t)|\, dt$

(b) $\Delta_v(a, b) \ge \int_a^b |v(t)|\, dt$

(c) $\Delta_v(a, b) \le \int_a^b |v(t)|\, dt$

(d) None of these

(iii) The series $\sum x^n/n^2$ is uniformly convergent in the interval :

(a) $[0,1]$

(b) $[0,2]$

(c) $[-2,2]$

(d) $[-4,4]$

(iv) The Taylor remainder $\gamma_m(x)$, after m terms satisfies :

(a) $\lim_{x\to 0}\frac{\gamma_m(x)}{|x|^{m-1}}>0$

(b) $\lim_{x\to 0}\frac{\gamma_m(x)}{|x|^{m-1}}<0$

(c) $\lim_{x\to 0}\frac{\gamma_m(x)}{|x|^{m-1}}=0$

(d) None of these

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(v) For $x\in R$ the function $f'(x)=0$, then x is called :

(a) Stationary point

(b) Non stationary point

(c) $x=0$

(d) None of these

(vi) If $|U_n(x)|\le M_n, \forall n$ and $x\in X$, then the series $\sum U_n(x)$ will converges uniformly on X of the series $\sum M_n$ is converges. This test is known as :

(a) M-test

(b) Abel test

(c) Dirichet's test

(d) None of these

(vii) Let $f:S\subset R^2\to R$ be a function on an open set S of R^2 into R. Suppose that the partial derivatives $D_1 f$, $D_2 f$, $D_{12} f$, $D_{21} f$ and are continuous in S, then :

(a) $D_{12} f(a,b) = D_{21} f(a,b), (a,b) \in S$

(b) $D_{12} f(a,b) = D_{21} f(a,b), (a,b) \notin S$

(c) $D_{12} f(a,b) \neq D_{21} f(a,b), (a,b) \in S$

(d) None of these

Section-'B'

(Short Answer Type Questions) **5×2=10**

Note : Attempt all questions. Each question carries 2 marks.

2. Define the existence of Riemann Stiettzes integral.

Or

Let f be monotonic on $[a,b]$, then prove that f is Riemann Stiettjes integration.

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3. Let $V:[a,b] \to R^{12}$ be a curve if $C \in (a,b)$ then prove that

$$\Delta_v(a,b) = \Delta_v(a,c) + \Delta_v(c,b)$$

Or

Define the rectifiable curve and give the example.

4. State and prove the Weierstrass M-test for uniform convergence.

Or

Define point-wise convergence of a sequence of function give one examples.

5. State and prove chain rule.

Or

Define interchange of order of differentiation.

6. Define extremum problems with constraints.

Or

Explain the Lagrang's multiplies method.

Section-'C'

(Long Answer Type Questions) **5×5=25**

Note : *Attempt all questions. Each question carries 5 marks.*

7. State and prove the fundamental theorem of calculas.

Or

Let $f \in R[\alpha]$ on $[a, b]$ and $a < c < b$ the prove that $f \in R[\alpha]$ on $[a, c]$. Also prove that

$$\int_a^b f\, d\alpha = \int_a^c f\, d\alpha + \int_c^b f\, d\alpha$$

8. State and prove Riemann's rearrangement theorem.

Or

If v' is continuous on $[a, b]$ then prove that v is rectificable and

$$\Delta_v = \int_a^b |v'(t)|\, dt$$

PTO

9. State and prove Dirichtet's test for uniform convergence.

Or

State and prove Abel' test for uniform convergence.

10. State and prove inverse function theorem.

Or

State and prove Taylors theorem.

11. Find the shortest distance from the point $(3/2, 0)$ to the parabola $y^2 = 4x$.

Or

Determine the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 + \frac{3\sqrt{3}}{2} xy$$

Subject to the constraint $4x^2 + y^2 = 1$.