SE-35

M. A./M. Sc. (First Semester) Examination, Dec. 2022

(For Regular/Private/ATKT/Ex./Fail Students)

MATHEMATICS

Paper : Second

(Real Analysis)

Time Allowed : Three hours

Maximum Marks : 40 Reg./50 Pvt.

For direction of regular students :

All questions are compulsory. Section-A Objective type, attempt all the 5 questions are compulsory, each question carries 1 mark. Section-B Short answer type, attempt all the 5 questions are compulsory, each question carries 2 marks. Section-C Long answer type, attempt all the 5 questions are compulsory, each question carries 5 marks.

For direction of private students :

All question are compulsory. Section-A Objective type, attempt all the 5 questions are compulsory, each question carries 2 marks. Section-B Short answer type, attempt all the 5 questions are compulsory, each question carries 3 marks. Section-C Long answer type, attempt all the 5 questions are compulsory, each question carries 5 marks.

Section-'A'

(Objective Type questions)

1. Choose the correct option :

(i) If P_1 and P_2 are two partitions of $[a, b]$ then their common refinement is :

(a) $P_1 \cap P_2$

(b) $P_1 \cup P_2$

(c) $P_1 \times P_2$

(d) $P_1 + P_2$

(ii) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\ldots\ldots=$

(a) $\frac{1}{2}\log 2$

(b) $\frac{3}{2}\log 2$

(c) $\log 2$

(d) None

(iii) Weierstrass M-test applied to test of uniform convergence of :

(a) Series

(b) Sequences

(c) Product of two series

(d) None

(iv) A linear operator A on a finite dimensional vector space X is one to one if and only if the range of A is :

(a) onto

(b) into

(c) many one

(d) continuous

(v) If $u = x + y + z$, $v = x - y + z$, $w = x^2 + y^2 + z^2 - 2xyz$ then the value of $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ is :

(a) 1

(b) 0

(c) 2

(d) 3

Section-'B'

(Short Answer Type Questions)

2. Let $f : [a, b] \to R$ be bounded function and α be a monotonically increasing function. If P is any partition of $[a, b]$ then

$$L(P, f, \alpha) \leq U(P, f, \alpha)$$

Or

Give the definition and existence of Riemann Stieltjes integral.

3. Define closed curve and rectifiable curve.

Or

Define rearrangement of series with example.

4. Define uniform convergence, pointwise convergence and point of non uniform convergence.

Or

State and prove Weierstrass's M-test.

5. Explain linear transformation and linear operator.

Or

Let E be an open set in R^n, f maps E into R^m and $x \in E$ and

$$\lim_{h \to 0} \frac{|f(x+h) - f(x) - Ah|}{|h|} = 0$$

holds with $A = A_1$ and $A = A_2$ then show that $A_1 = A_2$.

6. Write a note on Lagrange's multiplier method.

Or

If $u_1 = \frac{x_2 x_3}{x_1}$, $u_2 = \frac{x_1 x_3}{x_2}$ and $u_3 = \frac{x_1 x_2}{x_3}$ then prove that $J(u_1, u_2, u_3) = 4$.

Section-'C'

(Long Answer Type Questions)

7. Let f be a bounded function and α be a monotonically increasing function on $[a, b]$ then $f \in R[\alpha]$ on $[a, b]$ if and only if for every $\epsilon > 0$ there exists a partition P such that

$$U(P, f, \alpha) - L(P, f, \alpha) < \epsilon$$

Or

If $f_1 \in R(\alpha)$, $f_2 \in R(\alpha)$ on $[a, b]$ then show that $f_1 + f_2 \in R(\alpha)$ and

$$\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1\, d\alpha + \int_a^b f_2\, d\alpha$$

8. State and prove Riemann's theorem.

Or

If γ be a continuously differentiable curve on $[a, b]$ then show that γ is rectifiable and

$$\Lambda_\gamma(a, b) = \int_a^b |\gamma'(t)| dt$$

9. State and prove Cauchy's general principle of uniform convergence.

Or

State and prove Tauber's theorem.

10. Let $f : E \subset R^2 \to R$ be a function on an open set E of R^2 into R. Suppose that partial derivatives $D_1 f$, $D_2 f$, $D_{12} f$ and $D_{21} f$ exists and are continuous in E then

$$(D_{12} f)x = (D_{21} f)x, \quad x \in E$$

Or

State and prove Taylor's theorem.

11. Find the largest and smallest distances from (0, 0, 0) to the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \qquad 0 < a < b < c$$

Or

Determine the maximum and minimum values of the function

$$f(x, y) = x^2 + y^2 + \frac{3\sqrt{3}}{2} x, y$$

Subject to the constraints

$$4x^2 + y^2 = 1$$