201050

M. Sc. (First Semester) Examination, Dec. 2021

(For Regular/ATKT/Ex. Students)

MATHEMATICS

Paper : Third

(Topology-I)

Time Allowed : Three hours

Maximum Marks : 42

Note : *Attempt questions of all three sections as directed. Distribution of marks is given with sections.*

Section-'A'

(Objective Type Questions) 7×1=7

Note : *Attempt all the following questions. Each question carries 1 mark.*

1. Choose the correct answer :

(i) The set of real number is :

(a) Countable

(b) Uncountable

(c) Finite

(d) None of these

(ii) In any topological space $A \cup D(A)$ is :

(a) Open

(b) Semi open

(c) Closed

(d) None of these

(iii) A constant function $f : X \to Y$ is always :

(a) Constant

(b) Continuous

(c) Discontinuous

(d) None of these

(iv) The limit point of $A = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\}$ is :

(a) 0

(b) 1

(c) ∞

(d) None of these

(v) A metric space is second countable if and only if it is :

(a) Regular

(b) Seperable

(c) Connected

(d) None of these

(vi) If X is connected then :

(a) X has many components

(b) X has no components

(c) X has only one components

(d) None of these

(vii) Γ ¬y components of a topological space is :

Open

(b) Closed

(c) Neither open nor closed

(d) None of these

Section-'B'

(Short Answer Type Questions) 5×2=10

Note : *Attempt all five questions. One qustion from each unit is compulsory. Each question carries 2 marks.*

Unit-I

2. Show that the set of intergers Z is countable.

Or

Prove that the subset of a countable set is countable.

Unit-II

3. Prove that a subset A of a topological space X is closed, iff $\overline{A} = A$.

Or

Prove that in a topological space, an arbitrary union of open sets is open.

Unit-III

4. State Kuratowski closure operator.

Or

Define continuity in topological space.

Unit-IV

5. Prove that second countable space is always first countable space.

Or

Define seperable spaces.

Unit-V

6. Prove that every indiscrete space is connected.

Or

Define locally connected spaces.

Section-'C'

(Long Answer Type Questions) 5×5=25

Note : *Attempt all the five questions. One qustion from each unit is compulsory. Each question carries 5 marks.*

Unit-I

7. State and prove Cantor's theorem.

Or

State and prove Schroeder-Bernstein theorem.

Unit-II

8. Let $(X, \mathfrak{J})$ be a topological space and let A be a subset of X. Then prove that :

(i) A° is an open set.

(ii) A° is the largest open set contained in A.

(iii) A is open iff $A^\circ = A$.

Or

Show that a subspace of a topological space is itself a topological space.

Unit-III

9. Prove that a mapping $f : X \to Y$ is continuous iff $\overline{f^{-1}(B)} \subset f^{-1}(\overline{B}) \ \forall \ B \subset Y$.

Or

Prove that a one-one onto map $f : (X, \mathfrak{J}) \to (Y, V)$ is a homomorphism iff $f(A^\circ) = [f(A)]^\circ \ \forall \ A \subset X$.

Unit-IV

10. Prove that a second countable space is always seperable.

Or

Prove that every subspace of a second countable space is second countable.

Unit-V

11. Prove that continuous image of a connected space is connected.

Or

Show that a topological space X is locally connected iff the components of every open subspace of X are open in X.

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