201115

M. Sc. (Third Semester) Examination, Dec. 2021

(For Regular/ATKT/Ex. Students)

MATHEMATICS

(Integral Transform-I)

Time Allowed : Three hours

Maximum Marks : 42

Note : *Attempt questions of all* ***three*** *sections as directed. Distribution of marks is given with sections.*

Section-'A'

(Objective Type Questions) **7×1=7**

Note: *Attempt all the following questions. Each question carries 1 mark.*

1. Write true/false against each questions :

(i) The value of L (cosh at) is equal to $\frac{a}{p^2+a^2}$ for $p > a$.

(ii) Integral Transformed defined by

$$f(p) = \int_{-\infty}^{\infty} K(pt)F(t)\,dt$$

the function $K(pt)$ is called Shibling theorem.

(iii) One dimensional wave equation's

$$\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial y}{\partial t}$$

(iv) An equation of the form

$$\int_0^t \frac{F(u)\,du}{(t-u)^{\alpha}}$$

is called Abels form.

(v) Two dimensional heat conductors is

$$\frac{\partial u}{\partial t} = \frac{1}{k}\nabla^2 u$$

(vi) The solution of two dimensional Laplaces equation is plane polar co-ordinates r and θ

$$\left(Ar^{n}+Br^{-n}\right)\exp(\pm in\theta)$$

(vii) If $\left(D^{2}+1\right)y=0$ and $y=1, D(y)=0$ when to then $y(t)$ is $\cos t$.

Section-'B'

(Short Answer Type Questions) 5×2=10

***Note:** Attempt all questions. Each question carries 2 marks.*

2. Show that

$$L^{-1}\left\{\frac{P^{2}}{P^{2}+4a^{2}}\right\}=\frac{1}{2a}$$

$(\cosh at \sin at + \sinh at \cos at)$

Or

Find the value of

$$L^{-1}\left\{\log\frac{P+3}{P+2}\right\}$$

3. Use the convolution theorem find

$$L^{-1}\left\{\frac{P^{2}}{\left(P^{2}+4\right)^{2}}\right\}$$

Or

Solve

$$\left(D^{2}+m^{2}\right)x=a\cos nt,\ t>0$$

4. Write the equation of Laplace of two dimensional in polar and cylindrical co-ordinates.

Or

Write short notes on wave equation.

5. Find the general solution of following equation

$$\frac{\partial^{2}u}{\partial t^{2}}=c^{2}\frac{\partial^{2}u}{\partial x^{2}}$$

Or

Find the bounded solution of

$$\frac{\partial u}{\partial t}=\frac{\partial^{2}u}{\partial x^{2}}$$

where $y(0, t) = 1$, $y(x, 0) = 0$

6. Write short notes Heat conduction equation.

Or

A beam which is hinged at its ends $x = 0$ and $x = l$ carries a uniform lead w_0 per unit length. Find the deflection any point.

Section-'C'

(Long Answer Type Questions) 5×5=25

Note : *Attempt all questions. Each question carries 5 marks.*

7. It $F(t)$ is piecwise continuous and satisfies $|F(t)| \le Me^{at}$ for all $t \ge 0$ for some constants a and M. then

$$L\left\{\int_0^t F(x)\,dx\right\} = \frac{1}{p} L\{F(t)\}$$

Or

Prove that

$$\int_0^\infty e^{-x^2} = \frac{\sqrt{\pi}}{2}$$

8. Show that

$$\int_0^\infty \cos x^2\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

Or

Solve

$$(D^2 + D)y = t^2 + 2t$$

where $y(0) = 4$ and $y'(0) = -2$

9. Find the solution of the wave equation.

$$\frac{\partial^2 u}{\partial t^2} = h^2 \frac{\partial^2 u}{\partial x^2}$$

if the string is originally plucked at the middle point by giving it an initial displacement the mean position.

Or

A string is stretched between two fixed points (0, 0) and (c, 0). If it is displaced into the curve $y = b\sin\left(\frac{\pi x}{a}\right)$ and released from rest in that position at time $t = 0$. Find its displacement at any time $t > 0$ and at any point $0 < x < c$.

10. Find the temperature $u(x,t)$ in a bar whose ends $x=0$ and $x=l$ are kept at temperature zero and whose initial temperature

$$f(x)=\sin\left(\frac{\pi x}{l}\right).$$

Or

Solve the equation

$$F(t)=1+\int_0^t F(u)\sin(t-u)\,du$$

and verify your solution.

11. To find the temperature $u(x,t)$ in a bar length l which in perfectly insulated also at the ends at $x=0$ and $x=l$ assuming that the initial temperature of bar is $u(x,0)=\psi(x)$.

Or

Solve the boundary value problem

$$\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}-g,\ \ x>0,\ \ t>0$$

with the boundary condition

$$u(x,0)=0=u_t(x,0),\ x\geq 0$$

$$u(0,t)=0,\ \lim_{x\to\infty}u_x(x,t)=0,\ t\geq 0$$