Printed Pages – 7

201114

M. Sc. (Third Semester) Examination, Dec. 2021

(For Regular/ATKT/Ex. Students)

MATHEMATICS

(Integration Theory-I)

Time Allowed : Three hours

Maximum Marks : 42

Note : *Attempt all questions. Marks are distributed against the sections.*

Section-'A'

(Objective Type Questions) **7×1=7**

Note : *Attempt all questions. Each question carries 1 mark.*

1. Choose the correct answer :

(i) The Lebesgue measure on $[0,1]$ is a :

(a) σ-finite measure

(b) Finite measure

(c) Infinite measure

(d) None of the above

(ii) Each σ-finite measure is a ....

(a) Signed measure

(b) Semifinite

(c) Monotonic

(d) None of the above

(iii) If μ and ν are signed measures such that ν is both absolutely continuous and singular with repsect to μ, then ν = ...

(a) -1

(b) 1

(c) ∞

(d) 0

(iv) If f is a measureable function, then :

(a) Its positive part and negative part is measurable.

(b) Only positive part is measurable

(c) Only negative part is measurable

(d) None of the above

Give True / False :

(v) Let (X, M, μ) be a measurable space then $\mu(A) \leq \mu(B)$.

(True/False)

(vi) A step function is not a measureable function.

(True/False)

(vii) Hahn Decomposition is not unique.

(True/False)

Section-'B'

(Short Answer Type Questions) **5×2=10**

Note : *Attempt all **five** questions. Each question carries 2 marks.*

2. Give the formal definition of measureable set A. And prove that if $\mu^*(E) = 0$, then E is Lebesgue measurable.

Or

Show that the union of two measureable sets is also measureable.

3. Show that each σ-finite measure is saturated.

Or

Define σ-finite and semi finite measurable.

4. Define comptetion, measure and prove that an open set in a metric space is measurable.

Or

Show that, if f is a measurable function, then $|f|$ is also measurable but converse is not true.

https://www.mcbuonline.com

5. Let μ be a signal measure and $\langle E_n \rangle$ be a disjoint sequence of measurable sets such that

$$\left| \mu \left(\bigcup_{n=1}^{\infty} E_n \right) \right| < \infty$$

then prove that the series

$$\sum_{n=1}^{\infty} \mu (E_n)$$

is absolutely convergent.

Or

Define signed measure.

6. Define positive and negative set.

Or

Define Total Variation and Mutually Singular measure.

Section-'C'

(Long Answer Type Questions) 5×5=25

***Note :** Attempt all questions. Each question carries 5 marks.*

7. Let (X, M, μ) be a measure space. If $E_i \in M$, $\mu(E_i) < \infty$ and $E_i \supset E_{i+1}$, then prove that

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu\, E_n.$$

Or

Prove that the union of finite number of measurable sets is also measurable.

8. Show that a function is meaurable iff its positive and negative parts are measurable.

Or

Define σ-Algebra and outer measure. Prove that the class of measureable sets is a σ-ring.

9. Let C be a constant and f and g are two measurable real valued functions defined on the same domain. Then show that $f+g$, $f \cdot g$, $g-f$ and if are also measurable.

Or

Show that a function is measurable if and only if the set $\{ x : f(x) < r \}$ is measurable for every rational number.

10. Show that the signed measure ν is finite only if $|\nu|$ is finite.

Or

State and prove Hahn's Decomposition theorem.

11. State and prove Jorden decomposition theorem.

Or

Define variation of signed measure. If E and F are measurable set and μ is signed measurd such that $E \subset F$ and $|\mu(f)| < \infty$, then show that $|\mu(E)| < \infty$.