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211128

M. Sc. (Fourth Semester) Examination, June 2021

MATHEMATICS

Paper : IV (Gr-II : Optional)

(Advanced Special Function-II)

Maximum Marks : 42

Note: *Attempt questions of all three sections as directed. Distribution of marks is given with sections.*

Section-A

(Objective Type Questions) **7×1=7**

Note: *Attempt all the questions. Each question carries 1 mark.*

1. Choose the correct answer :

(i) The value of $J_{Y_2}(x)$ is :

(a) $\sqrt{\left(\frac{2}{\pi x}\right)}\sin x$

(b) $\sqrt{\left(\frac{2}{\pi x}\right)}\cos x$

(c) $\sqrt{\left(\frac{2}{\pi x}\right)}$

(d) None of these

(ii) The value of J_0' is :

(a) J_1

(b) $\dot{J}_1'$

(c) $-\dot{J}_1$

(d) None of these

(iii) The value of $P_n(1)$ is :

(a) 1

(b) 0

(c) –1

(d) None of these

(iv) The value of $P_2(x)$ is :

(a) $\frac{1}{2}(5x^2-1)$

(b) $\frac{1}{2}(3x^2-1)$

(c) $\frac{1}{2}(3x^3-1)$

(d) None of these

(v) If n is even, then the value of $\left[\frac{n}{2}\right]$ is :

(a) $-\frac{n}{2}$

(b) $\frac{(n-1)}{2}$

(c) $\frac{n}{2}$

(d) None of these

(vi) The value of $H_2(x)$ is :

(a) $4x^2 - 2$

(b) $4x^3 - 2$

(c) $4x^2 + 2$

(d) None of these

(vii) Simple Laguerre differential equation is :

(a) $xy'' + (1+x)y' + ny = 0$

(b) $xy'' + (1-x)y' + ny = 0$

(c) $xy'' - (1-x)y' + ny = 0$

(d) None of these

Section-B

(Short Answer Type Questions) **5×2=10**

Note: *Attempt all* ***five*** *questions.* ***One*** *question from each unit is compulsory. Each question carries 2 marks.*

Unit-I

2. Prove that :

$$J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cdot \cos x$$

Or

Prove that :

$$2J_0'' = J_2 - J_0$$

Unit-II

3. Show that :

$$nP_n = xP_n' - P_{n-1}'$$

Or

Show that :

$$P_n(-x) = (-1)^n P_n(x)$$

Unit-III

4. Prove that :

$$\int_{-1}^{1} P_m(x) P_n(x) dx = 0 \text{ if } m \neq n$$

Or

Prove that :

$P_n(0) = 0$, if n is odd

Unit-IV

5. Prove that :

$$H_n(-x) = (-1)^n H_n(x)$$

Or

Prove that :

$$H_n'(x) = 2n H_{n-1}(x)$$

Unit-V

6. Prove that :

$$L_n(0) = 1$$

Or

Prove that :

$$L_n(0) = n!$$

Section-C

(Long Answer Type Questions) **5×5=25**

Note: *Attempt all* ***five*** *questions.* ***One*** *question from each unit is compulsory. Each question carries 5 marks.*

Unit-I

7. Prove that :

$$2J_n'(z) = J_{n-1}(z) - J_{n+1}(z)$$

Or

Derive the Bessel differential equation.

Unit-II

8. Prove that :

$$(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$$

Or

Prove that :

$$P_n(x) = {}_2F_1\left[-n, n+1; 1; \frac{1-x}{2}\right]$$

Unit-III

9. Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials.

Or

Prove that :

$$\int_{-1}^{1} (x^2 - 1) P_{n+1}(x) P_n'(x)\, dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

Unit-IV

10. Prove that :

$$2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$$

Or

Derive the Rodrigues formula for $H_n(x)$.

Unit-V

11. Prove that :

$$x L_n'(x) = n\{ L_n(x) - L_{n-1}(x) \}$$

Or

Prove that :

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}\left(x^n e^{-x}\right)$$