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211132

M. Sc. (Fourth Semestr) Examination, June 2021

MATHEMATICS

(Optional Group-V)

(Integral Transforms-II)

Maximum Marks : 42

Note: *Attempt questions of all three sections as directed.*

Section-A

(Short Answer Type Questions)

Note: *Attempt all* ***five*** *questions.* ***One*** *question from each unit is compulsory. All questions carry equal marks.*

Unit-I

1. Solve : $(D^2+1)y = 6\cos 2t$, $y=0$ $Dy=1$ at $t=0$.

Or

A mass of 2 grams moves on the x-axis and is attracted towards origin with a force numerically equal to 8 x. If it is initially at rest at $x = 10$, find its position at any time t. Also a damping force numerically equal to 8 times the instantaneous velocity act.

Unit-II

2. An inductor of 3 henrys is in series, with a resistance of 30 ohms and an e.m.f. of 150 volts. Assuming that $t = 0$, the current is zero, find the current at time $t > 0$.

Or

A beam which is hinged at its end $x = 0$ and $x = l$ carries a uniform load w_0 per unit length. Find the deflection at any point.

Unit-III

3. Find the Fourier transform of $f(x)=\begin{cases} x & ; \ |x|\le a \\ 0 & ; \ |x|>a \end{cases}$

Or

Find Fourier sine transform of e^{-x}.

Unit-IV

4. If C_1 and C_2 are arbitrary constants, then prove that : $F\{C_1F(x)+C_2G(x)\}=C_1F[F(x)]+C_2F\{G(x)\}$

Or

If $F(x)$ has the Fourier transform $f(s)$, then prove that $F(x)\cos ax$ has the Fourier transform

$\frac{1}{2}[f(s-a)+f(s+a)]$

Unit-V

5. If $F\{F(x)\}=f(s)$, then prove that : $F\{F'(x)\}=is\,f(s)$.

Or

Find finite Fourier cosine transform of $f(x)=x,\ 0<x<\pi$.

Section-B

(Long Answer Type Questions)

Note: *Attempt all **five** questions. **One** question from each unit is compulsory. All questions carry equal marks.*

Unit-I

6. Find the solution of $\dfrac{\partial u}{\partial t}=K\dfrac{\partial^2 u}{\partial x^2}$

which tends to zero as $x\to\infty$ and which satisfy the boundary value conditions

$u=f(t)$, when $x=0,\ t>0$

$u=0$, when $x>0,\ t=0$

Or

Solve boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \qquad (x > 0,\ t > 0)$$

where

$$\left.\begin{aligned} u(x, 0) &= 0 \\ u_t(x, 0) &= 0 \end{aligned}\right\} x > 0$$

$$\left.\begin{aligned} u(0, t) &= F(t) \\ \lim_{x \to \infty} u(x, t) &= 0 \end{aligned}\right\} t \geq 0$$

Unit-II

7. An alternating e.m.f. $E \sin wt$ is applied to an inductance and a capacitance C in series. Show that the current in the circuit is $\frac{Ew}{(n^2 - w^2)L}(\cos wt - \cos nt)$, where $n^2 = \frac{1}{EC}$.

Or

A beam which is clamped at its ends $x = 0$, $x = l$ carries a uniform load w_0 per unit length. Show that the deflection at any point is $y(x) = \frac{W_0 x^2 (l - x)^2}{24\,EI}$.

Unit-III

8. Find the Complex Fourier transform of $f(x) = e^{-a|x|}$, where $a > 0$ and x belongs to $(-\infty, \infty)$.

Or

Find the Fourier sine and cosine transform of x^{m-1}.

Unit-IV

9. State and prove the convolution theorem for Fourier transform.

Or

State and prove the Parseval's identity for Fourier series.

Unit-V

10. Find the finite cosine transform of

$$f(x) = \left(1 - \frac{x}{\pi}\right)^2, \quad 0 < x < \pi$$

Or

Find finite Fourier sine and cosine transform of $f(x) = x^2, \ 0 < x < 4$.