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PG-20416

TERM END EXAMINATION – 2020
M. Sc. FINAL YEAR
MATHEMATICS
Integral Transforms with Applications

[Maximum Marks: **70**

Note *: Time – According to University Timing.*
All questions are compulsory. All questions carry equal marks.

1. (a) Write the first shifting property of Laplace transform. Hence show that if $L\{F(t)\} = \frac{1}{p} e^{-1/p}$ then

$L\{e^{t}F(3t)\} = \frac{e^{-3/(p-1)}}{(p-1)}$ **[7]**

(b) If $L^{-1}\left\{\frac{p}{(p^2+1)^2}\right\} = \frac{1}{2} t \sin t$,

then find $L^{-1}\left\{\frac{18p}{(9p^2+1)^2}\right\}$ **[7]**

<u>OR</u>

(a) Define convolution of two functions F(t) and G(t) of class A. Show that convolution of F(t) and G(t) is

$L^{-1}\{f(p).g(p)\}$ where $L^{-1}\{f(p)\} = F(t)$ and $L^{-1}\{g(p)\} = G(t)$

(b) Write Heaviside expansion formula.

2. (a) Using Laplace transform solve

$(D + 1)^2y = t$ given that y = - 3 when t = 0 and y = - 1 when t = 1. **[7]**

(b) Solve

$(D - 2)\, x + 3y = 0$

$2x + (D - 1)\, y = 0$

Where x(0) = 8 and y(0) = 3 **[7]**

OR

(a) State and prove Abel's Integral equation.

(b) Solve the integral equation using Laplace transform -

$F(t) = t^2 + \int_0^t F(u).\sin(t - u)du$

3. (a) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$. Hence find Fourier sine transform of $\frac{1}{x}$. **[7]**

(b) Use the Fourier sine inversion formula to obtain f(x), if

$\tilde{f}_s(p) = \frac{p}{1+p^2}$ **[7]**

OR

(a) Using Parseval's identity, prove that

(i) $\int_0^\infty \frac{ds}{(s^2+1)^2} = \frac{\pi}{4}$

(ii) $\int_0^\infty \frac{s^2}{(s^2+1)^2} ds = \frac{\pi}{4}$

(b) Find the finite sine transform of f(x) = cos Kx

4. (a) Write the Elementary properties of the Henkel Transform. **[7]**

(b) Prove: **[7]**

$M\left[\int_0^x f(u)du : p\right] = \frac{-1}{p}, f * (p+1)$

OR

(a) Write the relation between Fourier and Hankel transform.

(b) Find the Hankel transform of

$$f(x) = \begin{cases} a^2 + x^2 \ , & 0 < x < a \quad n = 0 \\ 0 & , x > a \quad n = 0 \end{cases}$$

5. (a) A tightly stretched string with fixed end points x = 0 and x = ℓ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{\ell}\right)$.

If it is released from rest from this position, find the displacement y (x, t). **[7]**

(b) Write one dimensional heat conduction equation with example. **[7]**

OR

(a) By the use of Fourier transform solve the equation-

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Under the condition

u = 0 at x = 0

$$u = \begin{cases} 1 & , 0 < x < 1 \\ 0 & , x \geq 1 \end{cases}$$

When t = 0

and u is bounded.

(b) Find the solution of the equation-

$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2}$$

Which tends to 0 (zero) as x→∞ and which satisfies the condition

u = f(t) when x = 0, t > 0

and u = 0 when x > 0, t = 0.

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