## PG-20413

## TERM END EXAMINATION - 2020 <br> M. Sc. FINAL YEAR <br> MATHEMATICS <br> Integration Theory and Functional Analysis

[Maximum Marks: 70

Note : Time - According to University Timing.
All questions are compulsory. All question carry equal marks.

1. (a) Let $F$ be a closed subset of $X$. Then $F$ is a locally compact Hausdorff space and the Baire sets of $F$ are those sets of the form $B \cap F$, where $B$ is a Baire set in $X$. Thus if $F$ is a closed Baire set, the Baire subsets of $F$ are just those Baire subsets of $X$ which are contained in F. Prove that the Borel sets of $F$ are those Borel sets of $X$ which are contained in $F$.

## OR

(b) Explain continuous functions with compact support with example.

## OR

(c) Let $\mu$ be a finite measure defined on a $\sigma$-algebra M which contains all the Baire sets of a locally compact space $X$. If $\mu$ is inner regular, then prove that it is regular.
2. (a) State and prove Radon-Nikodym Theorem.

## OR

(b) State and prove Riesz Representation theorem.

## OR

(c) State and prove Fubini's Theorem.
3. (a) Prove that every normed linear space $X$ is isomorphic to a linear manifold in the second conjugate space $X^{* *}$.

## OR

(b) Explain Quotient space of a normed linear space with example.

## OR

(c) State and prove uniform boundedness principle.
4. (a) State and prove Hahn-Banach theorem for complex linear space.

## OR

(b) Show that a normed linear space is locally compact iff it is finite dimensional.

## OR

(c) Show that if a normed space X is reflexive, then it is complete.
5. (a) Let $X$ be an inner product space and $x, y \in X$, then prove

$$
|<x, y>| \leq\|x\|\|y\|
$$

## OR

(b) Prove that an operator T on a Hilbert space H can be uniquely expressed as $T=A+i B$, where $A, B$ are self adjoint operator on $X$.

## OR

(c) State and prove Paseval's identity. . XX .

